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A GAS EJECTOR SYSTEM AND A DIFFERENTIAL EJECTOR

V. A. Malanichev UDC 533.697.5

1. Introduction. A theoretical investigation is conducted on the efficiency of using a system of gas ejectors with cylindrical mixing chambers and the limiting case of this system – the differential ejector. Mixing is examined for gases with equal stagnation temperatures and identical physical characteristics. The process of mixing gases in a differential ejector was first investigated in [1], where an error in the solution of the system of equations led to the loss of one condition of optimizing each stage of the differential ejector.

Here this error is corrected and the solution to the problem of a differential ejector is presented.

The transition from a single-stage ejector with a cylindrical mixing chamber to a system of sequential ejectors with cylindrical mixing chambers (Fig. 1) can improve the characteristics of a single stage ejector. The improvement is possible for two reasons. First, the differential mixing process can prolong the formation of the critical regime [1-3], which leads to a more efficient operation of the ejector. Second, differentiation increases the number of variable parameters in the ejector design, which can improve the efficiency of the mixing process. Here we investigate the effect only of the last factor; that is, it is assumed that the critical regime does not prevent optimization of the mixing process in each of the ejectors of the system. This approach is correct, because the effect of forming the critical regime is practically uncoupled with the ejector design specifics [1].

2. Optimization Criteria for a Single-Stage Ejector. We will examine mixing in an ejector with a cylindrical mixing chamber for two gases with identical physical characteristics c_p, κ, and stagnation temperature T_0. The total pressures are p_{01} and p_{02}; the mass flow rates are G_1 and G_2, where $p_{01} < p_{02}$. The gases are totally mixed in the chamber and there are no losses. In this case, the laws of conservation of mass flows, momenta and energies for a cylindrical mixing chamber are [4]

$$p_{0m} = \frac{1}{q(\lambda_m)\left(\frac{\gamma_1}{p_{01}q(\lambda_1)} + \frac{\gamma_2}{p_{02}q(\lambda_2)}\right)}; \tag{2.1}$$

$$z(\lambda_m) = \gamma_1 z(\lambda_1) + \gamma_2 z(\lambda_2), \tag{2.2}$$

where $z(\lambda) = \lambda + 1/\lambda$, $q(\lambda) = \lambda\left(1 - \frac{\varkappa - 1}{\varkappa + 1}\lambda^2\right)^{1/(\varkappa-1)}$, $\gamma_1 = G_1/(G_1 + G_2)$; $\gamma_2 = G_2/(G_1 + G_2)$, and p_{0m} is the total pressure of the gas mixture. From Eq. (2.2) it follows that for given values of the reduced velocities λ_1 and λ_2 there are two values of the reduced velocity of the gas mixture λ_m. One of these corresponds to the subsonic velocity of the gas mixture ($\lambda_m = \lambda_{m\ell} < 1$) and

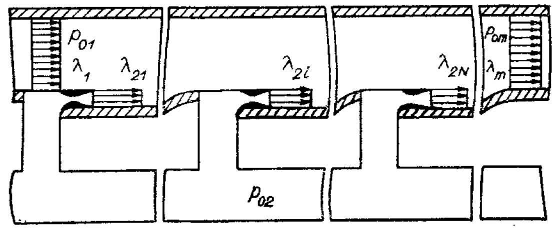

Fig. 1

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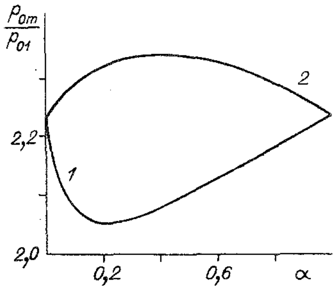

Fig. 2

the other to the supersonic velocity of the gas mixture ($\lambda_m = \lambda_{mr} > 1$). It is known that for $\lambda_m = \lambda_{m\ell}$, the following condition

$$\lambda_1 = 1,\ p_{02}\pi(\lambda_2) = p_{0m}\pi(\lambda_m) \tag{2.3}$$

is fulfilled when the ejector operation is optimized [1], where $\left(\pi(\lambda) = \left(1 - \frac{\varkappa - 1}{\varkappa + 1}\lambda^2\right)^{\varkappa/(\varkappa-1)}\right)$. The value of $p_{0m\ell}$, which corresponds to this condition, and λ_m are determined from the system (2.1) and (2.2). For $\lambda_m = \lambda_{mr}$, the optimum operation of the ejector corresponds to the condition

$$\lambda_1 = \lambda_*,\quad \lambda_2 = \lambda_* \tag{2.4}$$

$\left(\lambda_* = \sqrt{(\varkappa + 1)/(\varkappa - 1)}\right)$. Here

$$p_{0m} = \frac{1}{\left(\frac{\gamma_1}{p_{01}^{(\varkappa-1)/\varkappa}} + \frac{\gamma_2}{p_{02}^{(\varkappa-1)/\varkappa}}\right)^{\varkappa/(\varkappa-1)}}. \tag{2.5}$$

Physically, the subsonic value $\lambda_{m\ell}$ corresponds to operating the ejector with an ideal subsonic diffuser, and the supersonic value λ_{mr} to operating the ejector with an ideal supersonic diffuser. By ideal, we mean the ability to establish the total flow pressure in the diffuser without losses.

3. The Optimum Ejector System. In the ejector system, the gas mixture from the preceding stage is one of two working gases in the following stage. From Eq. (2.1) it follows that

$$dp_{0m}/dp_{01} > 0,\ dp_{0m}/dp_{02} > 0.$$

Therefore in the optimum ejector system each ejector should operate in the optimum regime.

We now examine the operation of a system of two ejectors with subsonic flow of a mixture of gases in the end of the mixing chamber of each ejector. Let the low-pressure gas G_1 enter the system in two parts: $G_{11} = \alpha G_1$ in the first injector and $G_{12} = (1 - \alpha)G_1$ in the second. A typical dependence of the total pressure of the gas mixture for the system of ejectors in this case $p_{0m}(\alpha)/p_{01}$ is shown in Fig. 2 (curve 1). The calculation was done with $p_{02}/p_{01} = 50$, $\gamma_1/\gamma_2 = 1$, and $\kappa = 1.4$. From the calculated results it follows that the mass separation of the low-pressure gas decreases the total pressure of the mixture compared to a single-stage ejector. Let the high-pressure gas G_2 enter the system in two parts: $G_{21} = \alpha G_2$ in the first injector and $G_{22} = (1 - \alpha)G_2$ in the second. A typical dependence $p_{0m}(\alpha)/p_{01}$ for this case is also shown in Fig. 2 (curve 2). It can be seen that the mass separation of the high-pressure gas increases the total pressure of the mixture. It can be shown that when two gases with identical physical characteristics and identical stagnation temperatures are mixed, only the separation of the input of the high-pressure gas, independent of the ratios p_{02}/p_{01} and γ_1/γ_2, leads to an increase of the total pressure of the gas mixture. This proof is omitted because of its complexity. If we also replace one of the two ejectors by two ejectors, we obtain a further increase in the total mixture pressure in the ejector system.

Thus the problem of optimizing a system with an arbitrary number of ejectors is reduced to determining the optimum method of dividing the high-pressure gas among the ejectors in the system. In a system with N ejectors, this problem is solved numerically by the method of descent in the space of the (N − 1)-th coefficient of the specific mass flow of the gas

$\alpha_k = G_{2k}/G_2$, $k = 1,\ldots,N$, where $\sum\limits_1^N \alpha_k = 1$. For a supersonic velocity of the gas mixture, the transition from the optimum single-stage ejector to a system ejector does not change the total pressure of the mixture, because the mixing of the gases occurs with a rate λ_* in each ejector. This can be shown with the use of Eq. (2.5).

4. Differential Ejector. The differential ejector is the limiting case of a system of N ejectors. Here the amount of high-pressure gas fed into each stage tends to zero and the number of stages increases to infinity, such that the total amount of gas fed to the ejector is finite. Then in each stage of the elementary ejector

$$\gamma_1 = 1, \quad \gamma_2 = dG_2 \Bigg/ \left(G_1 + \int\limits_0^G dG_2 \right).$$

In the transition to a differential ejector, the condition (2.3) takes the form

$$\lambda_1 = 1, \quad p_{02}\pi(\lambda_2) = p_{01}\pi(\lambda_1), \tag{4.1}$$

that is, in each stage the static pressure of the high-pressure gas is equal to the static pressure of the low-pressure gas, which moves at the speed of sound. If the ejector operation is optimized, the system of Eqs. (2.1) and (2.2) are transformed to

$$\frac{\pi'(\lambda_2)}{\pi(\lambda_2)} d\lambda_2 + \frac{q''(1)}{2q(1)} (d\lambda_m)^2 + \frac{dG_2}{G_1 + \int\limits_0^G dG_2} \frac{q(1)\pi(\lambda_2)}{q(\lambda_2)} + O(dG^2) = 0; \tag{4.2}$$

$$\frac{z''(1)}{2} (d\lambda_m)^2 = (z(\lambda_2) - 2) \frac{dG_2}{G_1 + \int\limits_0^G dG_2} + O(dG^2). \tag{4.3}$$

We introduce the concept of the specific mass flow of the high-pressure gas $n = \int\limits_0^G dG_2/G_1$.

Then the basic equation for the differential ejector follows from the system (4.2) and (4.3)

$$\pi'(\lambda_2)d\lambda_2/[\pi(\lambda_2)\varkappa(\lambda_2 - 1)] = dn/(1 + n). \tag{4.4}$$

Hereafter, the reduced velocity of the high-pressure gas λ_2, the total mixture pressure p_{0m}, and the area of the mixing chamber F_m will be taken as functions of the specific mass flow n: $p_{0m}(n)$, $\lambda_2(n)$, and $F_m(n)$. Equation (4.4) is solved by the method of separation of variables. After integrating, we obtain

$$(\lambda_* + \lambda_2)^{1/[(\varkappa-1)(\lambda_*+1)]} (\lambda_* - \lambda_2)^{1/[(\varkappa-1)(\lambda_*-1)]} (\lambda_2 - 1) = \frac{C_1}{1+n}. \tag{4.5}$$

The constant C_1 is determined for n = 0 by the value of $\lambda_2(0)$, which for (4.1) has the form $\lambda_2(0) = \pi^{-1}\{\pi(1)\cdot p_{01}/p_{02}\}$. The function

$$\varphi(\lambda) = (\lambda_* + \lambda)^{1/[(\varkappa-1)(\lambda_*+1)]} (\lambda_* - \lambda)^{-1/[(\varkappa-1)(\lambda_*-1)]} (\lambda - 1)$$

is a monotonically increasing function of its argument and increases from zero to infinity as λ grows from unity to the maximum value λ_*. Consequently, to each value of the function $\phi(\lambda)$ there corresponds a single value of the argument λ. Then p_{0m} in the differential ejector for given p_{02}, p_{01}, and $n_0 = G_2/G_1$ from (4.5) can be determined by calculating the value of $\lambda_2(n_0)$ and substituting it into (4.1); that is

$$p_{0m} = p_{02}\pi(\lambda_2(n_0))/\pi(1). \tag{4.6}$$

From the form of Eqs. (4.5) and (4.6), and also from the properties of the functions $\phi(\lambda)$ and $\pi(\lambda)$, it follows that the total mixture pressure increases with increasing specific mass flow n_0 of the high-pressure gas.

We now determine the equation for changing the area of the mixing chamber in the differential ejector. In each elementary ejector, the differential area dF_m consists of two terms: $dF_m = dF_m^+ + dF_m^-$. The first term is equal to the area of the high-pressure gas nozzle $dF_m^+ = \pi(\lambda_2) F_m dn/[\pi(1) q(\lambda_2)(1+n)]$. The second is equal to the constriction of the mixing chamber after complete gas mixing, area of which is required to drive the gas mixture through the input into the following ejector at the speed of sound:

$$dF_m^- = F_m \frac{q''(1)\,dn}{q(1)\,z''(1)\,(1+n)}\,(z(\lambda_2)-2).$$

Then

$$\frac{dF_m}{F_m} = \frac{dn}{1+n}(1-\varkappa(\lambda_2-1)). \tag{4.7}$$

From (4.7) it can be seen that for $\lambda_2 > (\kappa + 1)/\kappa$ the area of the mixing chamber decreases for an optimum differential ejector, but increases for smaller values of λ_2. As follows from (4.1) and (4.5), $\lambda_2(0)$ is determined by the pressure ratio p_{02}/p_{01} and the specific mass flow n. For $p_{02}/p_{01} > \pi(1)\Big/\left[\pi\left(\frac{\varkappa+1}{\varkappa}\right)\right]$ the mixing chamber initially constricts with increasing n, and then expands. As a result of solving Eq. (4.7), we find

$$F_m = \frac{C_2(\lambda_* - \lambda_2)^{1/[(\varkappa-1)(\lambda_*-1)]}}{\pi(\lambda_2)(\lambda_* + \lambda_2)^{1/[(\varkappa-1)(\lambda_*+1)]}(\lambda_2-1)}.$$

Here the constant C_2 is determined for n = 0 by the values $\lambda_2(0)$ and $F_m(0)$.

Figure 3 shows the results of calculating the total pressure of the gas mixture in an optimum system of ejectors $p_{0m}(n)/p_{01}$ for $p_{02}/p_{01} = 50$ and $\kappa = 1.4$. Curve 1 corresponds to a single-stage ejector, 2 to a system of five ejectors, and 3 to a differential ejector.

5. Theory of the Differential Ejector [1]. Section 2 shows the results of solving for the parameters of an optimum ejector in the general case of an arbitrary flow of high-pressure gas into one stage. In going to a differential flow of the high-pressure gas, the optimization criteria can be reduced to a simpler form, which was done in obtaining the condition (4.1). But a qualitative change in the optimization criterion can not take place, as was obtained in [1], because the condition $\lambda_1 = 1$ for the optimum stage is not included in [1]. Consequently, there is an error in the limiting transition. Now we will show what it is.

First we note that instead of Eq. (2.2), which in the notation of [1] has the form

$$(1+n+\delta n)z(\lambda_m) = (1+n)z(\lambda_1) + \delta n z(\lambda_2),$$

where δn is the amount of the high-pressure gas entering the elementary stage, in [1] the equation $\delta[(1 + n)z(\lambda_1)] = \delta n z(\lambda_2)$ was actually used. Thus, those solutions were excluded where the supersonic flow at the mixing chamber input can correspond to subsonic flow of the gas mixture which is formed by gas mixing at the shock front; that is, only the class of continuous functions $\lambda_1(n)$ was examined.

The actual error is as follows: in the system (2.1) and (2.2), initially the pressure function of the mixture $p_{0m}(\lambda_1, \lambda_2, n + \delta n)$ is expanded in a series in δn:

$$p_{0m}(\lambda_1, \lambda_2, n+\delta n) = p_{0m}(n) + A(\lambda_1, \lambda_2)\delta n + O(\delta n^2),$$

and then it is asserted that the conditions for an extremum in the function $p_{0m}(\lambda_1, \lambda_2, n + \delta n)$ coincide with the conditions for an extremum in the function $A(\lambda_1, \lambda_2)$ with an accuracy on the order of δn. In the general case for an arbitrary function, this approach is not valid. This can easily be seen using the function $y(x, \delta n) = (1 + x\cdot\delta n)^2$ as an example. Therefore, the validity of this approach must be proven in each actual case. We will show that the optimization conditions of each ejector stage was lost namely as a result of this error in the limiting transition.

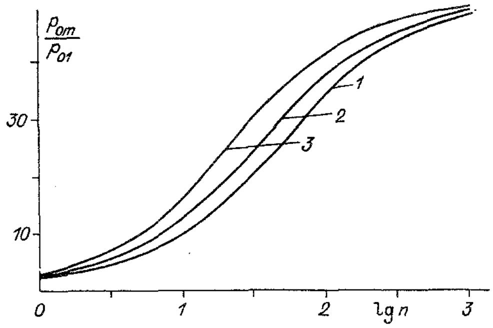

Fig. 3

Let two bounded positive functions f(x) and g(x), which have continuous second-order derivatives, be given in the region [0, a] (a > 1), where

$$g'(1) = f'(1) = 0, \quad g''(1) \neq 0 \neq f''(1). \tag{5.1}$$

And let the function $\varepsilon(x, y, \delta n)$ be given, which is defined by the following system of equations

$$\varepsilon(x, y, \delta n) = \frac{1 + \delta n}{\left(\frac{1}{g(x)} + \frac{\delta n}{g(y)}\right) g(z)}; \tag{5.2}$$

$$(1 + \delta n) f(z) = f(x) + \delta n f(y) \tag{5.3}$$

where δn is a small parameter. The curves of the conditional extrema are determined by the equations $\partial\varepsilon(x, y, \delta n)/\partial x = 0$ for

$$\frac{\varepsilon(x, y, \delta n)}{\left(\frac{1}{g(x)} + \frac{\delta n}{g(y)}\right)} \left(\frac{g'(x)}{g^2(x)} - \frac{g'(z) f'(x)}{g^2(z) f'(z)}\right) = 0 \tag{5.4}$$

and $\partial\varepsilon(x, y, \delta n)/\partial y = 0$ for

$$\frac{\varepsilon(x, y, \delta n)}{\left(\frac{1}{g(x)} + \frac{\delta n}{g(y)}\right)} \left(\frac{g'(y)}{g^2(y)} - \frac{g'(z) f'(y)}{g^2(z) f'(z)}\right) = 0, \tag{5.5}$$

where the values of $\varepsilon(x, y, \delta n)$ and z are found from (5.2) and (5.3). From Eqs. (5.4) and (5.5) it follows that the lines x = 1 and y = 1 are curves of conditional extrema of the function $\varepsilon(x, y, \delta n)$.

In the limiting transition $\delta n \to 0$, in the system (5.2) and (5.3), we obtain directly that

$$\delta\varepsilon(x, y) = \delta n \left[1 - \frac{g(x)}{g(y)} - \frac{(f(y) - f(x))}{g(x)} \frac{g'(x)}{f'(x)}\right].$$

Thus, in order for the line x = 1 to satisfy the condition $\partial\delta\varepsilon(x, y)/\partial x = 0$, it is necessary that

$$(g'(x)/f'(x))'|_{x=1} = 0. \tag{5.6}$$

From the limitations (5.1) it follows that in the neighborhood of the point x = 1, the functions g(x) and f(x) can be expanded in the series

$$g(x) = g_0 + g_2(x-1)^2 + g_3(x-1)^3 + o((x-1)^3),$$
$$f(x) = f_0 + f_2(x-1)^2 + f_3(x-1)^3 + o((x-1)^3).$$

Therefore Eq. (5.6) is equivalent to

$$g_2/g_3 = f_2/f_3. \tag{5.7}$$

Only in this case can the conditional extrema of the function $\varepsilon(x, y, \delta n)$ be obtained from the main term of the expansion of $\varepsilon(x, y, \delta n)$ in terms of the parameter δn. If $g(x) = q(\lambda)$ and $f(x) = z(\lambda)$, then the condition (5.7) is not fulfilled. Therefore, in the limiting transition $\delta n \to 0$, there is a loss of condition (4.1) for optimizing each stage of the differential ejector.

In conclusion, we note that in the case where the combined operation of the diffuser and the ejector is optimized by having a subsonic gas mixture flow out the ejector $\lambda_{1opt} < 1$, in [1] it is proposed that the velocity of the low-pressure gas be maintained at λ_{1opt} as an optimization criterion at the input to each elementary ejector. This is not valid. Because the diffuser is located only after the final stage, λ_{1opt} can be used only at the output of the final stage. Therefore it is sufficient to make the mixing process nonoptimum ($\lambda_1 \neq 1$) only in one or a few of the final stages.

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DYNAMIC STRAIN OF A CONDUCTING HALF SPACE WITH A CAVITY IN A STRONG MAGNETIC FIELD

L. A. Fil'shtinskii and L. I. Fomenko

UDC 539.3

The mechanical excitation of dia(para)magnetics in a static magnetic field creates an induced (rotational) current inside the body, which leads to the formation of Lorentz body forces, which are calculated by a tensor of Maxwellian stresses, which introduce large corrections in the stress state of the body.

Below we examine a conducting elastic half space with tunnel cavities which is subjected to mechanical excitation in a homogeneous static magnetic field. The corresponding magnetoelastic problem is reduced to a singular integral equation, which is solved numerically with the use of the method of mechanical quadratures. Calculated results are presented, which characterize the stress concentrations at the contour of the cavity as a function of the configuration of the aperture, the magnitude of the applied magnetic field, and the frequency of the excitation.

1. Basic Linear Magnetoelastic Equations and Formulation of the Problem. The total system of magnetoelastic equations include [1-3] the equations of motion

$$\partial_j\sigma_{ij} + \rho_e E_i + (\mathbf{j}\times\mathbf{B})_i = \rho\partial^2 u_i/\partial t^2 \quad (i, j = 1, 2, 3); \tag{1.1}$$

Maxwell's equations

$$\operatorname{rot}\mathbf{E} + \partial\mathbf{B}/\partial t = 0,\ \operatorname{rot}\mathbf{H} - \partial\mathbf{D}/\partial t = \mathbf{j},\ \operatorname{div}\mathbf{D} = \rho_e,\ \operatorname{div}\mathbf{B} = 0 \tag{1.2}$$

and the material equations

$$\begin{gathered}\mathbf{D} = \varepsilon\mathbf{E} + \alpha(\mathbf{v}\times\mathbf{H}),\ \mathbf{B} = \mu_e\mathbf{H} - \alpha(\mathbf{v}\times\mathbf{E}),\\ \mathbf{j} = \rho_e\mathbf{v} + \sigma(\mathbf{E} + \mathbf{v}\times\mathbf{B}),\ \alpha = \varepsilon\mu_e - \varepsilon_0\mu_0,\\ \sigma_{ij} = 2\mu\varepsilon_{ij} + \lambda\delta_{ij}\varepsilon_{kk},\ \varepsilon_{ij} = (1/2)(\partial_j u_i + \partial_i u_j),\\ \partial_i = \partial/\partial x_i,\ \mathbf{v} = \partial\mathbf{u}/\partial t \quad (i,\ j,\ k = 1, 2, 3).\end{gathered} \tag{1.3}$$

The boundary conditions on the separation surface between two media have the form

$$\begin{gathered}[\mathbf{E} + \mathbf{v}\times\mathbf{B}]_\tau = 0,\ [\mathbf{H} - \mathbf{v}\times\mathbf{D}]_\tau = 0,\\ [\mathbf{B}]_n = 0,\ [\mathbf{D}]_n = 0,\ [\sigma(\mathbf{E} + \mathbf{v}\times\mathbf{B}) + \rho_e\mathbf{v}]_n = 0,\\ [\sigma_{ij} + t_{ij}]n_j = X_{in} \quad (i, j, k = 1, 2, 3),\\ t_{ij} = E_iD_j + H_iB_j - (1/2)\delta_{ij}(E_kD_k + B_kH_k).\end{gathered} \tag{1.4}$$

Here $\mathbf{E}$, $\mathbf{D}$ and $\mathbf{H}$, $\mathbf{B}$ are the intensities and inductions, correspondingly of the electric and magnetic fields; ε, ε_0 and μ_e, μ_0 are the electric and magnetic permeabilities in the material and in a vacuum; ρ_e is the spatial density of the electric charge; $\mathbf{j}$ is the current density; ρ is the density of the material; u_i and σ_{ij} are the mechanical displacements and stresses; the X_{in} are the components of the external surface load; μ and λ are the Lamé constants; δ_{ij} is the Kronecker delta; and the symbol [ ] is a jump in the corresponding quantity at the separation line of the media.

Let a static magnetic field $\mathbf{H}^0$ act on a quiescent magnetoelastic medium. The external excitation creates a body strain and the creation of an electromagnetic field which can be

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